Site Selection Using Optimization Techniques

Vandana Bagla, Anjana Gupta

Abstract— The process of selection of sites for commercial activities involves myriad of qualitative, quantitative and financial factors. In general, there are multiple diversified factors. Due to the human tendency to depend more on emotions than reasons, there is every chance of reaching an irrational conclusion. This paper presents a formal system to evaluate comparative ranks of available sites for commercial activities and thereby to determine the long termed profitable decision using Lexicographic Approach. A versatile solution is provided to given problem using Weighted Penalty Method.

Index Terms— Analytic Hierarchy Process (AHP), Consistency, Eigen value, Eigen vector, Lexicographic Approach, Mixed Integer Programming, Weighted Penality Method .

1 Introduction

Almost all investment decisions involve multiple, diverse and complex set of social and financial factors which are quite hard to be overcome by mere intuition. The site selection process is an exemplification of an investment decision which involves the evaluation of attributes for maximization of profit and minimization of cost which are the most important requirements for the successful functioning of a particular business activity. Qualitative factors must also be considered while selection of a site for an activity.

A number of allocation problems have been solved in the recent past; for ready reference see Carlsson & Fuller[1], Ignizio[4], Ignizio & Cavalier [5], Serfini[7] and Azarm[8].

In this paper, we envision a site selection model for commercial activities which efficiently explores a multi-criteria decision-making model involving three objectives. Maximization of profit and minimization of set-up cost are the major objectives which are taken up as first and second objectives alternatively . Last but not the least objective is to rank various attributes such as capacity, neighborhood, connectivity, transport availability and proximity which are considerably important factors for a flourishing business.

The solution procedure consists of two phases. In the first phase, weights are allocated to various available sites based on various important aspects such as neighboring locality, broad or narrow connecting roads, area/capacity of the available sites, proximity factors such as competitive business rivals in the nearby areas, transport availability such as metro or other public transports. To accomplish this, an approach of Analytic Hierarchy Process (AHP) given by Saaty [10] is used.

In the second phase, the proposed problem seeking to maximize the profit, minimize the set-up cost and maximize the allotted weights, is modeled as mixed integer programming problem. Since the objective is to maximize the profit and weights. Also at the same time to minimize the cost, reversed costs are being taken after normalization. The problem is solved using hierarchical optimization method. The said problem is also be solved using Weighted Penalty Method developed by Prakash and Gupta[9].

The paper is organized as follows. Section 1 is introductory. Section 2 explains the Analytic Hierarchy Process (AHP). Section 3 describes the problem mathematically. Section 4 explains the proposed methodology to find the set of solutions. Section 5 illustrates the method via an example. The paper concludes in Section 6 for further application in real estate and many other important fields

2 Analytic Hierarchy Process (AHP)

The analytical hierarchy process (AHP) is a decision making approach designed to aid in the solution of complex multiple criteria problems in number of application domains. The outcome of AHP is a prioritized weighting of each decision alternative. The first step in the analytical hierarchy process is to model the problem as a hierarchy. The hierarchy is a structured mean of describing the problem at hand. It consists of an overall goal at the top level, a group of options or alternatives for reaching the goal and a group of factors or criteria that relate the alternatives to the goal. In most cases the criteria are further broken down into sub criteria, sub-sub criteria and so on in many levels as per the requirement of the problem. Once the hierarchy has been constructed, the participants use the AHP to establish priorities for all its nodes. In this, the elements of a problem are compared in pairs with respect to

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their relative impact on a property they share in common. The pair wise comparison is quantified in a matrix form by using the scale of Relative Importance given in Saaty [10] as shown in Table 1.

This scale has been validated for effectiveness, not only in many applications by a number of people, but also through theoretical comparison with a large number of other scales. During the elicitation process, a positive reciprocal matrix is formed in which $(i,j)^{th}$ element a_{ij} is filled by the corresponding number from the Table 1.

TABLE 1

Analytic Hierarchy Measurement Scale

Reciprocal measure of importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
3	Weak importance of one over another	Experience and judgment slightly favor one activity over another
5	Moderate importance	Experience and judgments moderately favor one activity over another
7	Strong Importance	An activity is strongly favored and its dominance is demonstrated in practice
9	Absolute Importance	The evidence favoring one activity over another is of the highest possible order of affirmation
2,4,6,8	Intermediate values between two adjacent judgments	When compromise is needed

The number is chosen according to the following criterion.

$$\begin{cases} a_{ij}, & \text{if } x_i \text{ dominates } x_j \\ 1/a_{ij}, & \text{if } x_j \text{ dominates } x_i \\ 1, & \text{if } x_i \text{ and } x_j \text{ do not dominate over one another} \end{cases}$$

The matrix so formed is called the reciprocal matrix. This reciprocal matrix is used to calculate the local priority weight of each criterion. The local priority weight (w) is the normalized eigen vector of the priority matrix corresponding to the maximum eigen value of the matrix. For detailed reasoning of this account we refer to Lunging [2], Ball & Srinivasan[3] , Bryson & Mobolurin[6] and Saaty[10].

An interesting property of the priority matrix is that if in addition its elements are such that

$$a_{ij}\, a_{jk} = a_{ik}, \qquad i \le j \le k \qquad (1)$$

then the derived priority vector **w** satisfies

$$w_i / w_j = a_{ij}, \qquad i < j \qquad (2)$$

Any reciprocal matrix satisfying (1) is called consistent. However in practice, the priority matrix seldom satisfies (1), thereby making it more important to define some relax measuring of consistency check, Saaty [10] introduced the concept of Consistency Index (CI) of a reciprocal matrix as the ratio $\frac{\lambda_{\max} - n}{n-1}$ where $\lambda_{\max}$ and n , respectively stand for the maximum eigen value and order of the reciprocal matrix.

The obtained CI value is compared with the Random Index (RI) given in Table 2. The table 2 had been calculated as an average of CI's of many thousands matrices of the same order whose entries were generated randomly from the scale 1 to 9 with reciprocal force. The simulation results of RI for matrices of size 1 to 10 had been developed by Saaty [10] and are given in Table 2.

TABLE 2

RANDOM INDEX (RI)

N	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

The ratio of CI and RI for the same order matrix is called the Consistency Ratio (CR).

3 PROBLEM DESCRIPTION AND MATHEMATICAL FORMULATION

Suppose a corporate body/M.N.C. deals in m different business/outlets and there are n available sites.

Problem is to allocate a suitable site out of the available sites to each business/outlet. While doing this, the two main objectives of the company are to maximize the overall profit and to minimize the overall cost. At the same time, the company wants to prioritize the sites carrying more weights.

Let p_{ij} $(i=1, \ldots,m;\ j=1,\ldots,n)$ denote the expected profit, when i^{th} business is set up on j^{th} site. Also let c_j be the overall cost and w_j be the weight of j^{th} site $(j=1, \ldots,n)$ calculated by AHP. Let ρ_j denote the normalized cost of the j^{th} site. Then $(1-\rho_j) = r_j$ denotes the reversed cost of the j^{th} site.

It is to be noted that we have reversed the cost as our objective is to minimize the cost and on the contrary, we are dealing with a maximization problem.

Then the above described model is formulated as the following three objective problem.

Maximize $Z(x) = (P(x), R(x), W(x))$

Where $P(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} p_{ij}\ x_{ij}$

$$R(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} r_j \; x_{ij}$$

$$W(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} w_j \; x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i=1, \ldots, m \qquad (1)$$

$$\sum_{i=1}^{m} x_{ij} \le 1, \quad j=1, \ldots, n \qquad (2)$$

$$x_{ij} \in \{0,1\}, \quad i=1 \ldots m; \; j=1, \ldots, n \qquad (3)$$

The constraint (1) ensures that each kind of outlet is allotted with a site. It is clear in constraint (2) that same site is not allotted for more than one outlet. In constraint (3) the value of x_{ij} is one if i^{th} outlet is allotted with j^{th} activity, otherwise x_{ij} is zero.

4 SOLUTION PROCEDURE

4.1 Allotting Weights Using AHP

In the site selection model, we construct hierarchy of attributes which are most important in decision making using AHP. To evaluate the hierarchy, various surveys are conducted to rate each attribute to others at the same level in a series of pair wise comparisons using a scale from 1 to 9 (Table 1).We rank each of the available site in the final set by evaluating the site with respect to upper level attributes separately as an illustrated in Table 4 and Table 5. The evaluation process finally generates the global weights for each available site of interest, as shown in Table 6 of the illustration.

4.2 Procedure to obtain optimal solution using Lexicographic Approach

Consider the three- objective linear programming problem

Maximize Z(x)= (P(x), R(x), W(x)) subject to given constraints.

The method requires that the objective functions are to be prioritized in decreasing order of importance. Let P(x) be the most prioritized and W(x) is the least prioritized objective. Then the method consists of following procedure.

Optimize the single objective problem consisting of P(x) as the objective function subject to given constraints. All other objectives are ignored.

Let P(x) = k_1 be the optimal solution obtained using integer programming in the first iteration.

Find the optimal solution of reconstructed single objective problem with R(x) as the objective function and an added constraint P(x) ≥ k_1 ,with the original constraint equations.

Let R(x) = k_2 be the optimal solution obtained in the second iteration.

Finally find optimal solution to the given problem in final iteration by reforming the problem as:

Maximize W(x)

Subject to

P(x) ≥ k_1

R(x) ≥ k_2

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i=1, \ldots, m \qquad (1)$$

$$\sum_{i=1}^{m} x_{ij} \le 1, \quad j=1, \ldots, n \qquad (2)$$

$$x_{ij} \in \{0,1\}, \quad i=1 \ldots m; \; j=1, \ldots, n \qquad (3)$$

The above stated procedure provides us with an optimal solution to the given three objective programming problem with prioritized objectives. A similar methodology may be adopted while considering cost as the first priority objective.

But the above stated model has its limitations. It does not provide alternatives to the aspirant which suits best to his pocket.

4.3 Procedure to obtain a set of efficient solutions using Weighted Penalty Method

Based on the feedback provided by the investor/corporate body/M.N.C, priorities are assigned to each of the three objectives. Here we have taken maximization of overall profit and minimization of cost as first and second priority objectives alternatively. Also maximization of qualitative ranks (weights) is assigned third priority.

This three-objective problem is reduced to an equivalent single-objective integer programming problem following the procedure developed by Prakash and Gupta [9]. Here p_{ij} denotes the expected annual profit if i^{th} business activity is set up at j^{th} site. Now we partition the set $\{p_{ij}$: i=1, …,m ; j=1,…,n$\}$ into the subsets L_k (k = -1, 1, …,q) in the following way.

L_{-1} consists of those p_{ij} for which i^{th} business activity can not be set up at j^{th} site. For example a petrol pump can not be set up in a multistoried shopping complex. Consequently we block allocation in that particular $(i, j)^{th}$ cell. Afterwards we follow the lexicographic arrangement of p_{ij}'s among the remaining p_{ij}. Let L_1 consists of those p_{ij} having the largest numerical value.

L_2 consists of p_{ij} having the next largest numerical value. Continuing in this way, finally L_q consists of p_{ij} having the smallest numerical value.

Now to deal with the cost function simultaneously, we calculate normalized cost ρ_j, for each potential site and consequently respective reversed cost $r_j = (1-\rho_j)$ for each available site.

Weights w_j have already been calculated for each site S_1, S_2,…S_n via AHP, as explained in section 4.1. Now Since the profit function P(x) is the first priority factor followed by cost function R(x) and then weight factor W(x). Assigning positive priorities $M_1, \ldots, M_q$, M_c, M_w, M_{-1} to each of the sum

$$\sum_{L_1} x_{ij}, \ldots, \sum_{Lq} x_{ij}, \quad \sum_{i=1}^{m}\sum_{j=1}^{n} r_j\ x_{ij}, \quad \sum_{i=1}^{m}\sum_{j=1}^{n} w_j\ x_{ij}, \quad \sum_{L_{-1}} x_{ij}$$

respectively.

Here $\sum_{L_k} x_{ij}$ is the sum of x_{ij}'s corresponding to p_{ij}'s belonging to L_k. Following points should be observed while allotting the priorities.

(i) No allocation can be made in set L_{-1}.

(ii) Cost factor has been reversed as our objective is maximization of the three given factors.

Now the priority weights assigned are

$M_1 >> M_2 >> M_3 \ldots >> M_q >> M_C >> M_w >> M_{-1}$

The symbol a >> b indicates *a* is arbitrarily large compared to *b*. Having done this, the problem with maximization of P(x) as the first priority objective, maximization of R (x) (minimization of C(x)) as the second priority objective and maximization of W(x) as the third priority objective, is reduced to a single objective integer programming problem.

$$\text{Maximize } Z(x) = \sum_{k=1}^{q} M_k \sum_{L_K} x_{ij} + M_c \sum_{i=1}^{m}\sum_{j=1}^{n} r_j\ x_{ij} + M_w \sum_{i=1}^{m}\sum_{j=1}^{n} w_j\ x_{ij} + M_{-1}\sum_{L_{-1}} x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i=1,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} \le 1, \qquad j=1,\ldots,n$$

$$x_{ij} \in \{0,1\}, \quad i=1, \ldots, m;\ j=1, \ldots, n$$

The optimal solution for the above problem yields the first efficient solution. Estimated annual profit of the business activities at the selected site is determined by adding the profits in the allocated cells. Minimum cost is calculated by adding the costs corresponding to allocated sites. Also corresponding total weights can be found out in the similar manner. Now to obtain second efficient solution, associate a cost M_{-1} (zero) with each of the variables x_{ij} for which p_{ij} is maximum and rest of the problem remains unchanged. This will somehow reduce the profit but at the same time reduce the cost born by the corporate body/ M.N.C. in general. Solve the resultant problem by adopting the same procedure.

The third and subsequent efficient solutions for the problem are obtained by repeatedly modifying the objective function and proceeding in the similar manner described above. This will provide a variety of solutions to the investor which suits best to his pocket.

Remark:- The proposed methodology provides an alternative to goal programming. As evident in goal programming, second and third priorities may be partially fulfilled.

5 ILLUSTRATION

We will now illustrate the proposed methodology via an example. Suppose a multi national company is seeking to invest in its different proposals like glossary stores, apparel outlets, gold outlets and petrol pumps. Each proposal requires a suitable site to be established. As a sample survey, Table 3 shows a set of available sites.

TABLE 3

BUISINESS PROPOSALS VERSES AVILABLE SITES

Business Proposals ↓	Available Sites →						
	S_1	S_2	S_3	S_4	S_5	S_6	S_7
B_1	3.5	3	4	2.5	3	1	1
B_2	3.5	3.5	2	1	1	1.5	1.5
B_3	2	3	3	1.5	2	2	3
B_4	4	-	2	2.5	2	1.5	-
Site Cost→	80	70	60	50	40	30	40

Here S_2 and S_7 are sites on upper levels of a multistoried shopping complex. Last row shows the costs of the available sites in requisite units. Intercellular costs show expected annual profit. For example the cell (1,1) shows that if B_1 proposal is setup on S_1 site, then annual expected profit is 3.5 units. The cells (4,2) and (4,7) are left blank because a petrol pump can not be set up on the upper levels of a multistoried building. The company wants an optimal set of solutions which should maximize the overall profit at the minimum cost which suits its pocket. At the same time the company does not want to compromise from quality point of view for the sake of its reputation and long term profitable business. To meet the requirement given by the M.N.C., all available sites were ranked as explained in section 4.1 by taking into account all important affecting attributes given in Fig. 1.

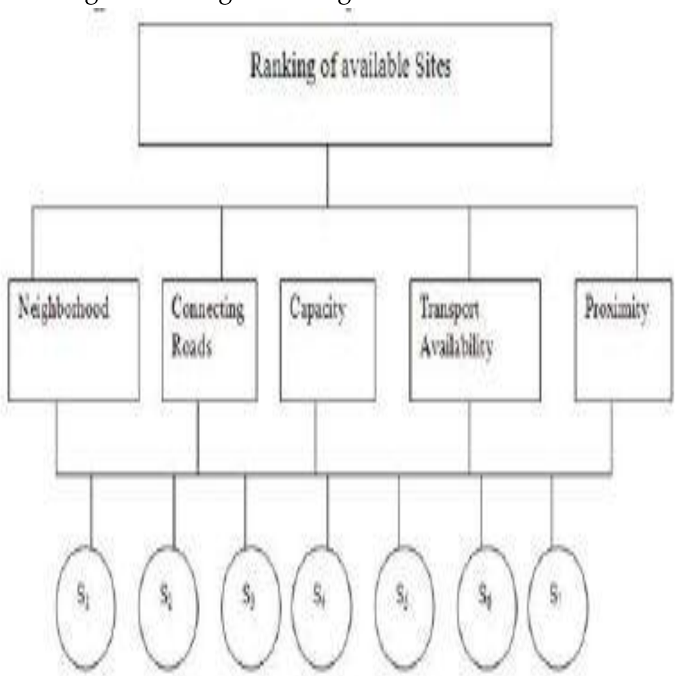

Fig. 1
Hierarchy for Site Selection

For this purpose a survey on thirty two people was conducted and reciprocal matrices were generated by taking mean of all values of matrices (having C.R < 0.1) for further calculations.

The important attributes were neighborhood, connecting roads, capacity, transport availability and Proxima. Neighborhood takes into account standard of the surrounding localities. Broad Connecting roads were given preferences. Capacity/Area of the available site also plays an important role. Availability of transport (Metro/Bus) is an important attribute for middle class people. Proxima plays a very important role in running of a successful business activity. Based on various surveys conducted on the available sites, weights were calculated for each attribute at each level. As an example Table 4 shows the comparison matrix and calculated weights at level two and Table 5 shows the comparison matrix for the neighborhood. Further more the global weights are calculated in Table 6.

TABLE 4
RANKING OF ATTRIBUTES

Ranking of attributes	Neighborhood	Connecting Roads	Capacity	Transport availability	Proximity	Weights
Neighborhood	1	1/5	1/3	3	1/7	.0633765
Connecting Roads	5	1	3	7	1/3	.261499
Capacity	3	1/3	1	5	1/5	.128976
Transport availability	1/3	1/7	1/5	1	1/9	.0333352
Proxima	7	3	5	9	1	.512813

λmax = 5.23748 C.I.= 0.0593688 C.R.=.0539164

TABLE 5
COMPARISON MATRIX FOR NEIGHBORHOOD

Neighborhood	S_1	S_2	S_3	S_4	S_5	S_6	S_7	Weights
S_1	1	4	2	9	3	7	5	.341144
S_2	1/4	1	¼	5	1/3	4	3	.0997997
S_3	1/2	4	1	8	3	6	5	.273792
S_4	1/9	1/5	1/8	1	1/7	1/2	¼	.0230014
S_5	1/3	3	1/3	7	1	5	4	.167413
S_6	1/7	1/4	1/6	2	1/5	1	1/3	.0339992
S_7	1/5	1/3	1/5	4	¼	3	1	.0608504

λmax = 7.44384 C.I.= 0.0739736 C.R.=.056041

TABLE 6
CALCULATION OF GLOBAL WEIGHTS FOR SITES

Sites	Neighborhood	Connecting Roads	Capacity	Transport Availability	Proxima	Global Weights
	.0633765	.261499	.128976	.0333352	.512813	
S_1	.341144	.0520361	.0919362	.390969	.0543163	.0880
S_2	.0997997	.40042	.224158	.059049	.0373937	.1611
S_3	.273792	.0913018	.403699	.24876	.404049	.3088
S_4	.0230014	.149872	.062503	.0332511	.0220899	.0611
S_5	.167413	.255662	.16271	.162048	.22726	.2204
S_6	.0339992	.0195063	.0208601	.0203357	.102307	.0631
S_7	.0608504	.0312011	.0341334	.0855867	.152583	.0975

Table 7 gives the reversed cost and calculated normalized weights for each site.

TABLE 7

CALCULATION OF REVERSED COST FOR SITES

Business↓	Available Sites →						
Proposals	S_1	S_2	S_3	S_4	S_5	S_6	S_7
B_1	3.5	3	4	2.5	3	1	1
B_2	3.5	3.5	2	1	1	1.5	1.5
B_3	2	3	3	1.5	2	2	3
B_4	4	-	2	2.5	2	1.5	-
Site Cost→	80	70	60	50	40	30	40
Normalized Cost(ρ_j)	.2162	.1892	.1622	.1351	.1081	.0811	.1081
Reversed Cost (r_j)	.7838	.8108	.8378	.8649	.8919	.9181	.8919
Normalized Weights (w_j)	.0880	.1611	.3088	.0611	.2204	.0631	.0975

First phase of the solution procedure has been accomplished using AHP.

5.1 Solution Using Lexicographic Approach

Now to solve the given problem using Lexicographic Approachexplained in section 4.2, taking maximization of profit as the single objective function subject to given constraints, the problem reduces to following integer programming problem.

Maximize P(x) = $3.5x_{11} + 3x_{12} + \ldots + 1.5x_{46}$

Subject to

$$\sum_{j=1}^{7} x_{ij} = 1, \; i=1,2,3,4$$

$$\sum_{i=1}^{4} x_{ij} \le 1, \; j=1,\ldots,7$$

$$x_{ij} \in \{0,1\}; \; i=1,\ldots,4 ; \; j=1, \ldots,7$$

Solving the above linear programming problem using integer programming, we get

$x_{13}=1$, $x_{22}=1$, $x_{37}=1$, $x_{41}=1$ yielding P(x) = 14.5.

Reconstructing the above problem in the second iteration as

Maximize R(x) = $0.7838x_{11} + 0.8108x_{12} + \ldots + 0.9181x_{46}$

Subject to

P(x) ≥ 14.5 and all above constraints in iteration 1.

Again solving the problem using integer programming, we get

$x_{13}=1$, $x_{22}=1$, $x_{37}=1$, $x_{41}=1$ yielding P(x) = 14.5

and R(x) =3.3243

Reforming the above problem in the third iteration as shown:

Maximize P(x) = $0.0880x_{11} + 0.1611x_{12} + \ldots + 0.631x_{46}$

Subject to

R(x) ≥ 3.3243 and all above constraints in iteration 2.

Solving above using integer programming

$x_{13}=1$, $x_{22}=1$, $x_{37}=1$, $x_{41}=1$ yielding P(x) = 14.5 ,

R(x) =3.3243 and W(x)=0.6554.

Table 8 shows the efficient solution by taking maximization of profit as first priority objective.

TABLE 8

PRIORITIZING MAXIMIZATION OF PROFIT

Iteration	Solution
1	$x_{13} = 1, x_{22} = 1, \; x_{37} = 1, \; x_{41} = 1$
2	$x_{13} = 1, x_{22} = 1, \; x_{37} = 1, \; x_{41} = 1$
3	$x_{13} = 1, x_{22} = 1, \; x_{37} = 1, \; x_{41} = 1$

Therefore the required optimal solution using Lexicographic Approachis given by

[Optimal profit = 14.5 units, Optimal cost = 250 units and Optimal weight = 0.6554]

Similarly the said problem is solved by the explained methodology taking minimization of cost as the first priority objective, maximization of profit as the second and maximization of weights as the third priority objectives. The efficient solution is given in Table 9.

TABLE 9

PRIORITIZING MINIMIZATION OF COST

Iteration	Solution
1	$x_{15} = 1, \; x_{24} = 1, \; x_{37} = 1, \; x_{46} = 1$
2	$x_{15} = 1, \; x_{26} = 1, \; x_{37} = 1, \; x_{44} = 1$
3	$x_{15} = 1, \; x_{26} = 1, \; x_{37} = 1, \; x_{44} = 1$

[Optimal cost = 160 units, Optimal profit = 10 units and Optimal weight = 0.4421]

5.2 Solution Using Weighted Penalty Method

Now to solve the above problem using Weighted Penalty Method, the reversed costs and normalized weights are calculated by adopting the similar procedure as explained before in section 4.1. we now provide priorities to respective profits viz. M1 to 4 units, M2 to 3.5 units, M3 to 3 units, M4 to 2.5 units, M5 to 2 units, M6 to 1.5 units and M7 to 1 unit as maximization of profit is the first priority objective. Also assigning Mc to respective reversed costs and Mw to normalized weights such that M1 >> M2 >> …>>M7 >>Mc >> Mw and adopting the procedure obtained in section 4.3, following set of values are obtained for various cells as shown in Table 10.

TABLE 10
Assigning Priorities Using Weighted Penalty Method

Sites → Proposals↓	S_1	S_2	S_3	S_4	S_5	S_6	S_7
B_1	$M_2+ .7838M_c+ .0880M_w$	$M_3+ .8108M_c+ .1611M_w$	$M_1+ .8378M_c+ .3088M_w$	$M_4+ .8649M_c+ .0611M_w$	$M_3+ .8919M_c+ .2204M_w$	$M_7+ .9181M_c+ .0631M_w$	$M_7+ .8919M_c+ .0975M_w$
B_2	$M_2+ .7838M_c+ .0880M_w$	$M_2+ .8108M_c+ .1611M_w$	$M_5+ .8378M_c+ .3088M_w$	$M_7+ .8649M_c+ .0611M_w$	$M_7+ .8919M_c+ .2204M_w$	$M_6+ .9181M_c+ .0631M_w$	$M_6+ .8919M_c+ .0975M_w$
B_3	$M_5+ .7838M_c+ .0880M_w$	$M_3+ .8108M_c+ .1611M_w$	$M_3+ .8378M_c+ .3088M_w$	$M_6+ .8649M_c+ .0611M_w$	$M_5+ .8919M_c+ .2204M_w$	$M_5+ .9181M_c+ .0631M_w$	$M_3+ .8919M_c+ .0975M_w$
B_4	$M_1+ .7838M_c+ .0880M_w$	M_{-1}	$M_5+ .8378M_c+ .3088M_w$	$M_4+ .8649M_c+ .0611M_w$	$M_5+ .8919M_c+ .2204M_w$	$M_6+ .9181M_c+ .0631M_w$	M_{-1}

The given integer programming problem reduces to
MaximizeZ(x)=(M2+0.7838Mc+0.0880Mw)x_{11}+
(M3+0.8108Mc+0.1611Mw)x_{12}
+…….+(M6+0.9181Mc+.0631Mw) x_{46}
Subject to

$$\sum_{j=1}^{7} x_{ij} = 1, \quad \mathbf{i = 1, 2, 3, 4}$$

$$\sum_{i=1}^{4} x_{ij} \le 1, \quad \mathbf{j = 1, \ldots, 7}$$

$x_{ij} \in \{0,1\}$; $i=1,\ldots,4$; $j=1, \ldots,7$

By solving the above problem, the set of efficient solutions prioritizing maximum profit is obtained as shown in Table 11.

TABLE 11
PRIORITIZING MAXIMIZATION OF PROFIT

Iteration	Solution	Profit	Cost	Weight
1	$x_{13}=1, x_{22}=1$, $x_{37}=1$, $x_{41}=1$	14.5	250	.6554
2	$x_{11}=1, x_{22}=1$, $x_{37}=1$, $x_{44}=1$	12.5	240	.4077
3	$x_{15}=1, x_{22}=1$, $x_{37}=1$, $x_{44}=1$	10.5	190	.6878
4	$x_{14}=1, x_{23}=1$, $x_{36}=1$, $x_{45}=1$	8.5	180	.6534
5	$x_{17}=1, x_{23}=1$, $x_{36}=1$, $x_{45}=1$	7	170	.6898
6	$x_{17}=1, x_{25}=1$, $x_{34}=1$, $x_{46}=1$	5	160	.4421

Similarly a set of efficient solutions is obtained by taking maximization of reversed cost (minimization of cost) as the first priority objective, maximization of profit as the second and maximization of weights as the third priority objective. Table 12 shows the set of efficient solutions by prioritizing minimization of cost.

TABLE 12
PRIORITIZING MINIMIZATION OF COST

Iteration	Solution	Cost	Profit	Weight
1	$x_{15}=1, x_{26}=1$, $x_{37}=1$, $x_{44}=1$	160	10	0.4421
2	$x_{13}=1, x_{25}=1$, $x_{37}=1$, $x_{44}=1$	190	10.5	.6878
3	$x_{13}=1, x_{22}=1$, $x_{34}=1$, $x_{41}=1$	260	13	.6190

5.3 Comparison of the Two Methodologies

Lexicographic Approach provides an optimal solution to the given problem according to the priorities assigned to objectives. But it does not provide choices to the aspirant which suit best to his pocket. Whereas using Weighted Penalty Method, a set of efficient solutions is obtained and investor has a wider choice. As evident from Tables 8 and 11, the optimal solution given by Lexicographic Approachby prioritizing maximization of profit is same as the first efficient solution obtained by Weighted Penalty Method. Same is the trend shown in Tables 9 and 12, while considering minimization of cost as the first priority objective. Also one may notice that the sixth efficient solution in Table 11 shows a profit of 5 units at a set up cost of 160 units and qualitative weights 0.4421. While at the same cost and weight, first efficient solution in Table 12 gains a profit of 10 crores. So the investor is provided with a lucrative option at the same set up cost.

6 *CONCLUSION*

While strategically managing our location decisions, we need solutions that address multiple logistics and economic factors involving real estate and our customers. Maximization of profit and minimization of set-up cost are the two major objectives. Third objective is to rank the sites qualitatively, for a flourishing business. The method also suggested that once starting up with a low profit/low cost model, high futuristic growth may be expected if qualitative ranks are high. For examplefifth efficient solution given in Table 11 and second efficient solution given in Table 12 gained highest ranks in spite of low cost and low profit, but the project may earn higher profits in near future in spite of low one time set up cost due to qualitative aspects. Present paper can be solved alternately by changing the level of priorities. For example high ranks may be prioritized to low cost etc. The proposed methodology may prove to be a powerful tool for efficient decision making. The model presented has potential application in the area of real estate, portfolio management, investment theory etc.

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